lines. The sound waves at $r/D=0$ are at least 15 dB lower than those with the cavity opened. At the highest pressure ratio of 2.7, it is seen that seeding the jet flow with intense sound waves results in a rapid decay in pressure level for a distance of 1 to 2 diam. For larger values of r/D, e.g., $r/D>5$, the effect of the cavity-generated waves is very small, whereas at the lowest pressure ratio of 2.22 the effect is quite pronounced, and the curves are separated by 10 dB or more.

The near field of the forward set of waves was studied by microphone traverses at $\theta=120°$. The sound pressure level at 12.5 kHz is plotted in Fig. 4. Comparison of this figure with Fig. 3 shows that the forward set of waves has relatively little interference from the sound field produced by the jet flow, except near the jet exit. There the nozzle exerts some influence, and the sound waves decay as spherical waves a short distance from the nozzle lip. The near field in the absence of the cavity-generated waves is shown in the same figure by the broken lines, and the intensities are substantially lower than those with the cavity width set at ½ in.

The sound pressure traverses of the downstream set of waves with the cavity closed are always higher than the forward set of waves for the four jet pressure ratios reported herein. This is to be expected, because the $\theta=45°$ traverses are closer to the noise-producing region of the jet. However, with the cavity opened, the sound pressure levels along the $\theta=120°$ traverses remains higher than the $\theta=45°$ traverses, since the drop in the sound pressure near the origin 0 is less rapid. This shows that the seeded sound has a greater influence in the forward direction, at least for this particular frequency. More experiments are needed to show whether this phenomenon is observed at other frequencies of the seeded sound waves, although traverses at 16 kHz reported in Ref. 6 show similar behavior, but the differences in the sound pressure level in the forward and downstream directions are not as large as those for 12.5 kHz.

Conclusions

Some preliminary results of the transmission of internally generated noise through a cold choked jet were presented. The sound source is an annular cavity cut into the inside surface of the nozzle. The amplitude and frequency of the cavity-generated waves can be controlled by adjusting the jet pressure ratio and the cavity width. Shadowgraphs taken of the radiated sound field show two sets of waves, one moving downstream and the other in the forward direction. They are separated by a region of poorly defined wave pattern, which is due to interactions between the two sets of waves. The radiated waves are quite spherical, and if the wave vectors of these two sets of waves are drawn, they will intersect on the jet boundary in the region where the acoustic beam emitting from the cavity leaves the jet.

Sound pressure traverses also were carried out for the downstream ($\theta=45°$) and forward ($\theta=120°$) sets of waves at a frequency of 12.5 kHz, with the cavity width set at ½ in. The $\theta=45°$ traverse shows some interference from the other noise sources in the jet such as shock cell noise, whereas the $\theta=120°$ traverse shows the waves to decay like spherical sound waves a short distance from the nozzle lip. For this particular frequency, the seeded sound waves have a larger influence in the forward direction, but more experiments are needed to show whether this is true for other frequencies as well.

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Correlation of Vibrational-Nonequilibrium Flow in Expansion Nozzles

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Introduction

MUCH work has been published on the vibrational-nonequilibrium flow in expansion nozzles.[1] Erickson[2] found numerically that the translational temperature T at large area ratios in the nonequilibrium flow calculations could be correlated as follows: the ratio of T to the temperature T_e at the same area ratio in equilibrium flow is independent of the area ratio and is given as a function of the stagnation temperature T_0 and the product $p_0\ell$, where p_0 is the stagnation pressures. The definition of ℓ follows. The purpose of this Note is to present an analytical verification of the above correlation.

Analysis and Results

We consider a pure diatomic gas flowing through a convergent-divergent nozzle with an area distribution given by

$$A/A^* = 1 + (x/\ell)^2 \tag{1}$$

where A is the cross-sectional area, x is the axial distance from the throat, $\ell = r^*/\tan\theta$, r^* is the throat radius, and θ is the asymptotic expansion angle. The asterisk denotes the conditions at the throat. In the range of the area ratio where vibrational freezing prevails, Eqs. (18) and (19) of Ref. 2 reduce to

$$dA/A = [(6T - 5T_0')/2T(T_0' - T)]dT \tag{2}$$

where T is the translational temperature, and T_0' is defined by

$$T_0' = T_0 + (2/7)\Theta(\sigma_0 - \sigma_f) \tag{3}$$

where Θ is the characteristic temperature for vibration, $\sigma_0 = [\exp(\Theta/T_0) - 1]^{-1}$ is the vibrational energy at T_0, and σ is the frozen vibrational energy. Both σ_0 and σ_f are dimensionless as a result of dividing by $R\Theta$. Where R is the gas constant. The solution of Eq. (2) is

$$\frac{A}{A^*} = K\left[\frac{T}{\Theta}\right]^{-5/2}\left[1 - \frac{T}{T_0'}\right]^{-1/2} \tag{4}$$

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where K is a constant. It should be noted that Eq. (4) holds true even if vibrational freezing occurs far upstream of the throat. If it occurs in the nozzle stagnation chamber, we have $\sigma_f = \sigma_0$, i.e., $T_0' = T_0$ from Eq. (3). Then we have, as is well-known,[3] $K = (5T_0/\Theta)^{5/2}/6^3$. In general, the constant K becomes a function of T_0 and $p_0\ell$.[4]

The equilibrium solution can be arranged as[2]

$$\frac{A}{A^*} = \frac{\beta C_0(\beta)}{tC_0(t)} \left[\frac{C_1(\alpha) - C_1(\beta)}{C_1(\alpha) - C_1(t)} \right]^{1/2} \tag{5}$$

where $\alpha = \Theta/T_0$, $\beta = \Theta/T_e^*$, $t = \Theta/T_e$, T_e is the equilibrium temperature,

$$C_0(t) = t^{-7/2}(\exp t - 1)^{-1} \exp[t \exp t/(\exp t - 1)] \tag{6}$$

and

$$C_1(t) = 7/t + 2/(\exp t - 1) \tag{7}$$

The constant β is determined as a function of α.[2] In the range of the area ratio where $\exp(-t) \ll 1$, Eq. (5) can be reduced to

$$\frac{A}{A^*} = K_e \left(\frac{T_e}{\Theta} \right)^{-5/2} \left(1 - \frac{T_e}{T_{0,e}'} \right)^{-1/2} \tag{8}$$

where

$$K_e = \beta C_0(\beta) \left[1 - \frac{C_1(\beta)}{C_1(\alpha)} \right]^{1/2} \tag{9}$$

and $T_{0,e}'$ is the value of T_0' for $\sigma_f = 0$.

Now the ratio of T to T_e at the same area ratio is obtained from Eqs. (4) and (8)

$$\frac{T}{T_e} = \left(\frac{K}{K_e} \right)^{2/5} C^{1/5} \tag{10}$$

where $C = [1 - (T_e/T_{0,e}')]/[1 - (T/T_0')]$. Equation (10) holds true in the range of the area ratio where not only the nonequilibrium flow is frozen but also the equilibrium temperature is so low and $\exp(-\Theta/T_e)$ can be neglected. These conditions can be easily satisfied at large area ratios, where, moreover $T_e/T_{0,e}'$ and T/T_0' are so small that the correction factor $C^{1/5}$ in Eq. (10) is close to 1. For example, if C deviates $\pm 10\%$ from 1, the factor $C^{1/5}$ deviates only $\pm 2\%$ from 1. Then, the correlation

$$T/T_e \approx (K/K_e)^{2/5} \tag{11}$$

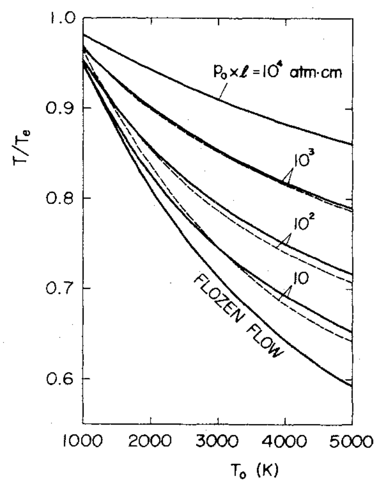

Fig. 1 Ratio of freestream temperature based on vibrationally relaxing nitrogen flow to that for equilibrium flow, correlated for large area ratios. ——— = present results: – – – – = results of Ref. 2.

holds to a close approximation. The right-hand side of Eq. (11) is a constant, which is a function of T_0 and $p_0\ell$. Other properties, e.g., the velocity u can be similarly correlated:

$$u/u_e = (T_0'/T_{0,e}')^{1/2} C^{-1/2} \tag{12}$$

However, the influence of area ratio does not disappear as it does with the temperature since $\pm 10\%$ deviation of C introduces $\mp 5\%$ deviation into $C^{-1/2}$ of Eq. (12).

The constant K can be easily determined by use of a sudden freezing approximation.[1] We use the criterion due to Phinney:[5]

$$\left| u \frac{d\sigma}{dx} \right|_{\text{equil.}} = \left(\frac{\sigma}{\tau} \right)_{\text{equil.}} \tag{13}$$

where $\sigma = (\exp t - 1)^{-1}$, and τ is the relaxation time. Substituting the equilibrium solution into Eq. (13) and using Eq. (1), we have

$$\frac{2[C_1(\alpha) - C_1(t)]^{1/2}[a(t) - 1]^{1/2}}{[1 - \exp(-t)]\,|a'(t)|} = \frac{(p_0\ell)C_0(t)}{(R\Theta)^{1/2}(p\tau)C_0(\alpha)} \tag{14}$$

where $a(t)$ denotes the right-hand side of Eq. (5) and $a'(t)$ denotes its first derivative with respect to t. We denote the solution of Eq. (14) by t_{sf}. Obviously, it is a function of T_0 and $p_0\ell$. The temperature at the sudden freezing point, T_{sf}, is

$$T_{sf} = \Theta/t_{sf}$$

Substitution of $\sigma_f = (\exp t_{sf} - 1)^{-1}$ into Eq. (3) yields T_0'. Now the value of the right-hand side of Eq. (4) at the sudden freezing point is determined except K. Substitution of t_{sf} into Eq. (5) yields the value of the area ratio at the sudden freezing point. Then, K can be determined from Eq. (4). T/T_e is then determined from Eq. (11) as a function of T_0 and $p_0\ell$.

Figure 1 shows the results obtained from Eq. (11) for nitrogen gas in comparison with the exact results of Ref. 2. In the present calculation, the values used for Θ and the dependence of τ on temperature were the same as those used in Ref. 2. For $p_0\ell = 10^4$ atm.cm, the difference between the two results is indistinguishable. Although it was assumed in Ref. 2 that equilibrium prevailed upstream of the throat, the present calculation for $p_0\ell \leq 10^2$ showed that the lower the stagnation temperature was, the farther upstream of the throat, sudden freezing occurred. Therefore, the results of Ref. 2 for $p_0\ell \leq 10^2$ should be somewhat corrected for lower T_0. In general, the present correlation resulting from the sudden freezing approximation agrees well with the results from the exact calculations.

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